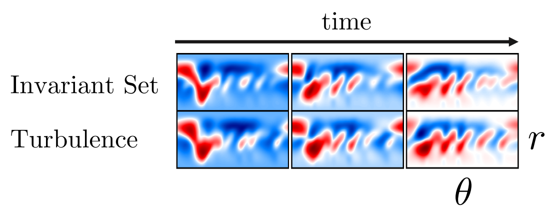
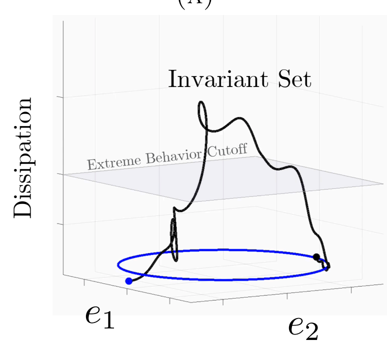
High-dimensional, non-linear dynamics are pervasive in science, impacting many frontiers of research from civil engineering [14] to plasma physics [15], active matter systems [12] to epidemiology [13], and beyond. Turbulence is often a "flagship" example of such systems, due to its ubiquity in climate, [2], industrial [1], and military applications [10]. Despite a rich history of study, predicting the behavior of turbulence has remained largely an unsolved problem and there exists an ongoing need for predictive and practical reduced order models of turbulence.

My Ph.D. research focused on making dynamical systems theory applicable to turbulence, often through the use of data-driven methods and reduced order models. In Axiom-A systems, where the Dynamical Systems Theory approach to chaos was developed, the chaotic state is always infinitesimally close to at least one invariant set in the phase space [11]. Since, for smooth flow maps, two trajectories that begin close will remain close for some finite interval of time, infinitesimal closeness implies *shadowing*. Shadowing is when two trajectories co-evolve (for some finite interval of time). It is very useful to identify instances where a chaotic state shadows elements of an invariant set, since the latter can be much simpler to describe and analyze.

However, infinitesimal closeness is not observed in turbulence on any reasonable timescale, partly due to the fact that only few invariant sets are known. There was a need for detecting shadowing without the notion of infinitesimal closeness. So, we developed a method that utilized a symmetry-informed projection of turbulent flow to detect intervals of time where shadowing occurs. An example of the events our method detects is illustrated in (A), which shows turbulence co-evolving with a trajectory embedded within an invariant set. Only a cross-section of the azimuthal component of velocity is plotted, but the entire flow fields are similar during shadowing. Our publications of this work [7; 5; 4] found elements from all known invariant sets to be shadowed and, despite only few invariant sets being known, found turbulence to shadow elements of these sets quite often.

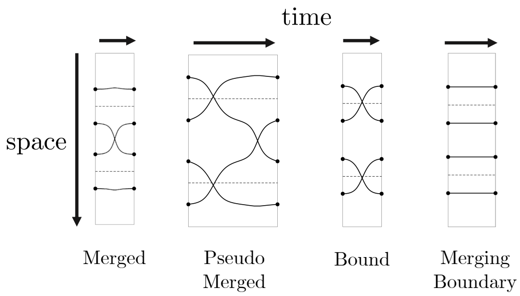


(A)

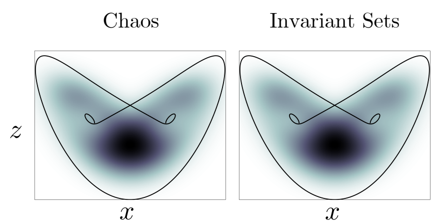


(B)

Shadowing is a very useful tool in practice. In an application to two-dimensional turbulence, I worked on uncovering a physical mechanism behind *extreme events*: statistically irregular, catastrophic behaviors of dynamical systems, such as oceanic rogue waves or sudden, intense precipitation. Using adjoint-based optimization methods, I determined an invariant set that characterizes this behavior (B) and used our symmetry-informed projection to show that turbulence shadowed elements of this invariant set during extreme events. It follows that, if one were able to identify shadowing in real time, the incoming extreme event could perhaps be forecast or, with some applied control, quelled entirely.



(C)



(D)

These invariant sets are also quite useful in understanding the statistics of chaotic flows, for instance, in studying the inverse energy cascade: an anomalous statistical flux of energy from small scales to large seen only in quasi two-dimensional systems. Vortex merging has long been been studied as a physical mechanism behind the inverse energy cascade [9], however, no one has yet been able to predict with precision when two vortices will merge [8]. Indeed, vortices are formally infinite dimensional objects, and it is difficult to construct an outcome boundary in such a large space. Using point vortices, I constructed a model with less than or equal to 5 degrees of freedom that well characterized the macroscopic outcome boundaries of vortex-vortex interactions. This model contains low-dimensional (invariant set) representations (C) of merged, bounded, and unbound vortex pairs, and even uncovered an intermediate pseudo-merged state that had been observed but not yet characterized in the literature [3]. Surprisingly, this reduced model predicted the merging boundary observed in experiment better than other theoretical results in the literature derived using extended flow fields. This work will soon be a first author publication (in preparation).

Long-time statistics are well understood in Axiom-A systems, where it is possible to compute the distribution of any chaotic observable as an expansion over *infinitely* many invariant sets [6]. However, as we saw with shadowing, this result relies heavily on the chaotic state venturing infinitesimally close to invariant sets and does not have a direct application to turbulence. To combat this, I developed a data-driven method that uses linear regression to approximate a Dynamical-Systems-Theory-like expansion using only a small number of invariant sets, and does not require the system coming close to them. (D) illustrates how well 21 invariant sets can approximate the coarse-grained likelihood of observing a given chaotic microstate of the system. This particular study shows that invariant sets are a powerful and seemingly quickly convergent representation of chaotic statistics. A publication is also in preparation.

As the above attests to, my PhD work at Georgia Tech has seen me through many different research projects, necessitating novel analytical skills and mastery over a number of numerical methods, integrators and solvers. While I began each of these projects with little to no prior exposure, I was able to produce novel contributions to the field in each project, resulting in either publishable papers or conference talks.

I am someone who can greatly contribute model reduction, machine learning and control oriented research problems—particularly in high-dimensional chaotic systems. Now that I have earned my Ph.D., I look forward to applying my expertise and making a positive impact. I welcome the opportunity to discuss my qualifications further and explore how I can contribute to your organization.

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